Homework #6 (100 points) - Show all work on the following problems:
(Grading rubric: Solid attempt = 50% credit, Correct approach but errors = 75% credit, Correct original solution = 100% credit, Copy of online solutions = 0% credit)

Problem 1 (25 points): Find the electric potential for the geometry of Example 3.3 if the electric potential on the boundary at $x=0$ is a step function with the constant value V_0 for $y = 0$ to $y = a/2$, and the constant value $-V_0$ for $y = a/2$ to $y = a$.

Problem 2 (25 points): Consider a rectangular pipe aligned parallel to the z-axis (similar geometry to Example 3.4), with three grounded metal sides ($V = 0$ for $y = 0$, $y = a$, and $x=0$), and one side held at a specified electric potential ($V = V_0(y)$ for $x = b$). Find a general series solution, including the formula for the coefficients, for the electric potential in the pipe.

Problem 3 (20 points): Consider a spherical shell with a constant electric potential V_0. Find the appropriate coefficient(s) for the series solution of Eq. 3.65 and determine the general solutions for the electric potential inside and outside the sphere. Verify that this matches the expected functional form (e.g. Example 2.7).

Problem 4 (30 points): Consider a spherical shell of radius R, with a specified electric potential on the surface of $V_0 = k \cos(3\theta)$. Find the electric potential inside and outside the sphere, and the charge density on the surface of the sphere. *Hint: First express the electric potential on the surface in terms of a sum of Legendre polynomials.*